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in the field of

STABILITY OF NONCONSERVATIVE SYSTEMS

submitted by

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During the period covered by the present progress report, several analytical studies were completed and several new studies have been initiated. As regards experimental work, the development of a set of demonstrational models has been completed and a report, as described more fully below, has been issued. Quantitative experimental work on one particular model has been initiated.

The completed studies may be summarized as follows:

1) One study is concerned with stability of equilibrium of nonconservative continuous systems with slight damping. More specifically, the stability of the equilibrium configuration of a linearly elastic solid (with or without internal damping) subjected to partial follower surface tractions (nonconservative loading) is investigated. Proportional loading is considered and it is proved that the flutter (oscillations with increasing amplitude) load parameter of the undamped system is an upper bound for that of the system with slight damping. The necessary and sufficient conditions for stability of the equilibrium state with respect to flutter-type motions, and sufficiency conditions for stability with respect to both divergence (or buckling) and flutter are established. Based on energy considerations, an approximate method for stability analysis is suggested which reduces to the usual energy criterion for the case of conservative loading.

This paper has been submitted for publication by S. Nemat-Nasser.

2) Another completed study, authored by J. Roorda and S. Nemat-Nasser, deals with the development of an energy method for stability

analysis of non-linear non-conservative systems. The purpose of this paper is to suggest an approximate method for finding the relationship between the force parameter and the amplitude of steady-state oscillations for non-linear, non-conservative, autonomous, dynamic systems. The method is based on the principle of conservation of energy; for the case of steady-state oscillations the energy dissipated over a cycle must equal the energy supplied by external sources. The stability of the system is discussed and an illustrative problem of a system with two degrees of freedom is worked out in detail.

3) In treating dissipative, dynamic systems, which are governed by nonself-adjoint linear operators, it is often found convenient to introduce the adjoint system (or field) and to consider formally a conservative process. The original field contains an energy sink and in the adjoint field an energy source of same strength is incorporated in order to make the combined field conservative. It is of interest to note that the notion of the adjoint field can be introduced also in treating nondissipative, nonconservative systems, i.e., dynamic systems subjected to circulatory forces. In particular, in structural systems subjected to follower forces the consideration of adjoint force fields leads to interesting consequences. Indeed, for this class of nonconservative systems both the original field and its adjoint force field are associated with energy sources, yet the combination of these two fields results in a conservative one.

An example illustrating the above considerations has been worked out and the study is scheduled for publication in an early issue of the AIAA Journal, authored by S. Nemat-Nasser and G. Herrmann.

4) The development of a fundamental set of models has been completed and described in a report, authored by G. Herrmann, S. Nemat-Nasser and S. N. Prasad. In this report nine different models, designed and constructed at the Structural Mechanics Laboratory of Northwestern University, are described, which serve the purpose of demonstrating instability of equilibrium of mechanical systems subjected to follower forces. Such forces are nonconservative, nondissipative and are also called circulatory forces. Instability is observed to occur by either divergence (static instability, attainment of another equilibrium position) or flutter (dynamic instability, oscillations with increasing amplitudes). The connection between some of these models and related nonconservative problems of elastic stability considered analytically in the past is indicated.

Several studies are currently in progress, as described below:

1) One study is concerned with the extent of asymptotic stability. When investigating the stability of the equilibrium configuration of a dynamic system, one may ask the following questions:

a) If the system which is initially in an equilibrium state is given an arbitrarily small disturbance, does it remain near the equilibrium state and in addition tend to return to the equilibrium? If it does the system is asymptotically stable. This is the problem of <u>local stability</u>.

b) What bounds must be placed on the magnitude of the initial disturbance so that, given a disturbance within these bounds, the system will eventually regain its equilibrium state? In other words, what is the extent of asymptotic stability? This is the problem of <u>stability in the large</u>.

It is obviously true that if the system is to be stable in the sense of (b) for some finite initial disturbance then it must be asymptotically stable in the local sense. Therefore one is concerned first with local stability and secondly, having established that, with the extent of stability.

In this study the latter problem is investigated, the system being assumed locally stable. In order to cause a loss of stability the initial disturbance, whatever form it takes, must be strong enough to enable the system to cross a stability boundary. The energy levels associated with such disturbances are studied and several theorems are derived which place sufficiency conditions on the allowable disturbances. Although most of the theorems are applicable to both conservative and nonconservative systems, their application to nonconservative problems is emphasized.

2) It was found that several different types of nonconservative problems of elastic stability can be put in like analytic form by the introduction of complex variables. It turns out that if this formulation is used, a number of interesting theorems regarding certain destabilizing effects can be established which throw additional light on the behavior of fluid-solid systems.

3) Quantitative experimental work on a mechanical system consisting of two articulated rigid rods with a rigid plate attached to the free end and subjected to an air jet, which is placed uniaxially

with the undeformed axis of the system, is in progress and some results have already been obtained in this rather delicate experimentation.

During the period covered by this progress report several papers and talks were presented covering the research carried out under the grant. In particular, a paper entitled "An Approximate Energy Method for Post-Critical Analysis of Nonconservative Systems," authored by S. Nemat-Nasser, J. Roorda and G. Herrmann, was presented at the Fifth U. S. National Congress of Applied Mechanics in Minneapolis, Minnesota, June 14-17, 1966. Further, a paper "Bending-Torsional Flutter of a Swept Wing in a High-Density, Low-Speed Flow" by S. N. Prasad, S. Nemat-Nasser and G. Herrmann was presented at the Fourth AIAA Aerospace Sciences Meeting in Los Angeles, California, June 27-29, 1966. Finally, a talk was presented by G. Herrmann at the Space Technology Seminar of the Stanford-Ames-ASEE-NASA Summer Faculty Institute at Stanford University on August 11, 1966. The title of the talk was, "Stability of Nonconservative Elastic Systems."